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A hydrodynamic model of super-deep penetration of a solid axisymmetric particle into a semi-infinite metallic target

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Abstract—Ten years investigation of the super-deep penetration problem allows one to understand the main features of the effect and prepare the foundation for a model that can describe the process. The paper is devoted to the creation of such a model taking account of real conditions of super-deep penetration process, such as an intensive pressure field which is generated in the target by the flux of powder particles accelerated by a special explosive accelerator. Accurate investigation of the problem allowed one to obtain a precise solution of the problem, at least for the cases of constant and time-dependent pressures. © 1997 Elsevier Science Ltd.

INTRODUCTION

The super-deep penetration effect (SDP) has already been investigated for more than 10 years [1–4], but the problem of constructing a correct model of the effect remains unsolved. The essence of the SDP effect is that on the interaction of a dense powder flux, generated by an explosive accelerator (Fig. 1) with a metallic target, the finite portion of these particles, about 0.1%, can penetrate the target to a depth, x, that is equal to $10^3 d$ or exceeds it, where d is the initial diameter of a particle. Such an effect can be observed for particles whose diameter is smaller than 10^{-4} m at the impacting velocity, U_o, varying within 1–3 km s^{-1} for various metallic targets, when the flux density ρ_f is equal to 10^3 kg m^{-3}, or exceeds it. This occurs despite the fact that the specific energy of such particles (0.5 MJ kg^{-1}) is insufficient for penetration to a depth greater than $50d$. This fact attracted the attention of researchers because it was impossible to explain by means of the existing models of penetration [5, 6].

Experimental investigation of SDP [1–4] allowed one to establish the most significant features of the effect:

- the effect occurs only in the case of a dense flux of particles, when $\rho_f \geqslant 250$ kg m^{-3}. The sizes of particles in SDP are also limited. For the flux accelerator used in [1–3] the ultimate diameter of particles should not exceed 10^{-4} m;
- zones of intense plastic deformation, produced by the particles moving in the target, were observed close to the axis of their motion. That part of these zones, which is located closest to the path of penetration, is found to be amorphous, i.e. the metallic material loses its crystalline structure [4].
- the motion of particles in the target in SDP depends on the mechanical and thermal properties of the target material, but is independent of the configuration of their grain boundaries [1, 2]. The channels formed by passing particles in the target during penetration close again after the particles have passed and can only be observed later on sections by means of a special etching and polishing technique.

The aim of the present paper is the creation of an exact hydrodynamic model of SDP which is an extension of the ideas stated in Refs [1–3].

CONDITIONS

An impulse pressure ($p \approx$ 10–15 GPa) generated on the collision of a flux of particles with a target [1, 2] makes it possible to consider the process occurring in that zone without allowance for the target strength [1–3]. As a rule, the velocity of particles does not exceed the speed of sound in a metal, $U_o < c$, for the Reynolds number $Re > 10^2$ [1, 2]. This allows one to describe the target material, interacting with a particle, as inviscid and incompressible.

The penetration of each separate particle under the SDP conditions can be divided into two stages: (1) an inertial period, when the particle velocity slows down under the action of hydrodynamic forces (the duration of this period is determined by both the time needed for the particle to penetrate the target to the depth $x = L$, where L is the particle length, and the time for the closing of the channel); (2) the motion of the particles after the closure of the channel which occurs as a result of the pressure generated by the flux of powder particles [1, 2].

NOMENCLATURE

c	speed of sound of a target [m s^{-1}]
C_f	drag coefficient of a particle
d	particle diameter [m]
dm_0	total mass of the target material upstream of the point O [kg]
dm_1	portion of the mass of the target material associated with jet '1' [kg]
dm_2	portion of the mass of the target material associated with jet '2' [kg]
dm_3	mass of the target material in front of the particle [kg]
H	ultimate depth of penetration [m]
l	distance between point O and rear surface of the particle [m]
L	particle length [m]
M	mass of a particle [kg]
O	point of convergence of the channel formed by a penetrating particle in the target
p and $p(t)$	pressure generated in the target by the flux of particles [Pa]
Re	Reynolds number
S	cross-sectional area of a particle [m^2]
t	current time [s]
t_c	time of channel convergence [s]
t_s	time needed for a particle to penetrate the target to the depth $x = L$ [s]
T	time period from the moment of channel convergence at the point O to the moment when jet '2' touches the rear side of the particle [s]
t_q	sum of time periods t_c, t_s and T, that determines the total duration of the first stage of penetration [s]
Δt	physically small time period [s]
U	velocity of particles in a fixed coordinate system or target material flow velocity in the coordinate system connected with the particle [m s^{-1}]
U_f	particle flux velocity [m s^{-1}]
U_q	particle velocity at the time $t = t_q$ [m s^{-1}]
U_{st}	stable particle velocity [m s^{-1}]
W	velocity of the channel 'walls' directed to the path of particles [m s^{-1}]
V	absolute value of the sum of the vectors of two velocities U and W
V_1	velocity of jet '1' originated from O and directed oppositely to direction of particle motion [m s^{-1}]
V_2	velocity of jet '2' originated from point O and directed along the direction of particle motion [m s^{-1}]
x	current depth of penetration of a particle [m]
x_q	depth of penetration at the time $t = t_q$
y, y_c, y_q and y_s	dimensionless parameters of calculations.

Greek symbols

α	angle of the convergence of a channel [rad]
μ	kinetic target viscosity [m^2 s^{-1}]
ρ	target density [kg m^{-3}]
ρ_f	flux density [kg m^{-3}].

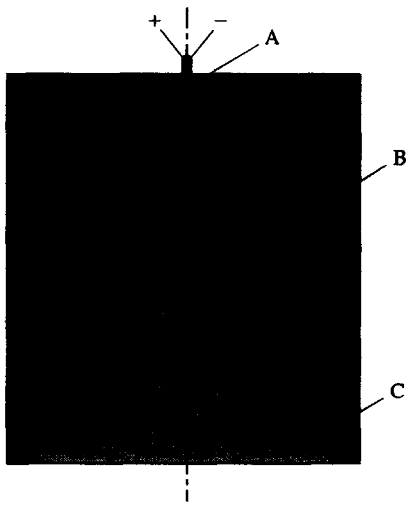

Fig. 1. A scheme of an explosive accelerator [1]: (A) detonator; (B) explosive charge; (C) powder container.

RETARDATION STAGE

In this stage the motion of a particle is determined by the hydraulic resistance of a target material and the inertia of the particle. When $Re \geqslant 100$ and it is not necessary to take into account the viscous share of the resistance force, the equation of motion can be written as

$$M\,dU/dt = -C_f \rho U^2 S/2 \qquad (1)$$

where M, U and S are the mass, velocity and cross-sectional area of the particle, respectively. C_f is the particle drag coefficient that depends on the particle shape, ρ is the target density and t is the time. Equation (1) holds only at large values of Re ($Re \gg 1$), otherwise it would be necessary to take account of the viscous component of the hydrodynamic resistance force. Under the initial condition $U = U_o$, when $t = 0$:

$$U = U_o \Big/ \left(1 + C_f \frac{\rho S U_o}{2M} t\right) \qquad (2)$$

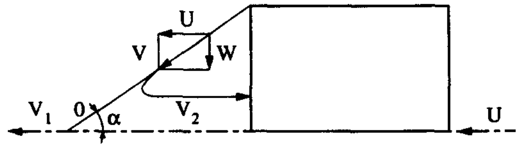

Fig. 2. Flow of target material near a particle in the coordinate system associated with the latter.

$$x = \frac{2M}{\rho S}\ln\left(1 + C_f\frac{\rho S U_o}{2M}t\right) \quad (3)$$

but as soon as x exceeds L the process changes substantially (Fig. 2).

SECOND STAGE OF PENETRATION UNDER A CONSTANT PRESSURE

Under the pressure p, generated by a flux of particles in the target, the streamlines of the target material flowing about the particle (all the further calculations will be performed in the coordinate system connected with a particle—SCP) turn, after the passage of the particle, to the axis of the particle motion through the angle α (Fig. 2). After the period of time

$$t_c = d/(2W) \quad (4)$$

the channel, formed in the target by the particle, collapses entirely at the point O separated from the rear part of the particle by the length $l = (d/2)/\tan\alpha$. Two jets originate from this point (Fig. 2). '1' moving with the velocity V_1 and directed along the initial flow and '2' moving with the velocity V_2 in the opposite direction. Since the velocity of the particle changes in the process of penetration, the coordinate system connected with the particle cannot be considered as inertial, but at any time one can point out such a physically small period of time Δt during which the change in the velocity is negligible and, consequently, both the particle and the point O in the SCP can be considered as fixed. The point O is the point at which streamlines turn, but the absolute value of the velocity at this point does not change [7, 8] because the pressure drop on the cavity walls remains the same during the whole process, $V_1 = -V_2 = V$ (Fig. 2). The mass of the liquid carried away by each of the two jets from the point O can be found from the laws of mass and momentum conservation:

$$dm_0 = dm_1 + dm_2$$
$$\cos\alpha V dm_0 = V dm_1 - V dm_2 \quad (5)$$

and

$$dm_1/dm_0 = (1+\cos\alpha)/2, \quad dm_2/dm_0 = (1-\cos\alpha)/2. \quad (6)$$

The velocity V can be determined using Bernoulli's law

$$0.5\rho V^2 = 0.5\rho U^2 + p. \quad (7)$$

For the time T, comparable with t_c, jet '2' overtakes the particle, retards on its rear surface and starts to push it.

During a physically small period of time Δt, determined earlier, the laws of mass and momentum conservation for the particle in the fixed coordinate system can be written as:

$$M dU = V dm_2 - U dm_3$$
$$dm_0 = dm_3 = \rho S U dt \quad (8)$$

where dm_3 and dm_0 are the masses of the liquid in the layer $U\Delta t$ in front and behind the particle, respectively. The use of eqns (6) and (7) yields

$$M dU/dm_0 - 0.5(1-\cos\alpha)V + U = 0. \quad (9)$$

Determining from Fig. 2

$$\cos\alpha = U/V = 1\Big/\sqrt{1+\frac{2p}{\rho U^2}} \quad (10)$$

and introducing the notation

$$y = \sqrt{2p/\rho U^2} \quad (11)$$

the equation of particle motion can be obtained

$$-(2M/\rho S)\sqrt{0.5\rho/p}\frac{\mathrm{d}y}{\sqrt{1+y^2}-3} = \mathrm{d}t \quad (12)$$

or, because $\mathrm{d}x = U\mathrm{d}t$,

$$-(2M/\rho S)\frac{\mathrm{d}y}{y(\sqrt{1+y^2}-3)} = \mathrm{d}x. \quad (13)$$

Calculation of eqns (12) and (13) is performed under the initial conditions $y = y_q$, when $t = t_q = t_s + t_c + T$. The time interval t_c is known, t_s is determined from eqn (3), when $x = L$. The unknown time interval T can be calculated from the equation:

$$l = \int_\tau^{t_q} V\mathrm{d}t \quad (14)$$

where $\tau = t_q - T$. Substituting l and V from eqn (7) into eqn (14) and performing its calculation, it is possible to obtain a transcendental equation for determining y and T

$$\sqrt{1+y^2} - \sqrt{1+y_s^2} = \rho S \mathrm{d}y_s/4M - \ln(y/y_s) \quad (15)$$

where

$$y_o = \sqrt{2p/\rho U_o^2}, \quad y_s = \sqrt{2p/\rho U_s^2},$$
$$U = U_o\Big/\left(1 + C_f\frac{\rho S U_o}{2M}\tau\right). \quad (16)$$

The solution of eqn (15) y_q is connected with T through

$$y_q = \sqrt{2p/\rho U_q^2} \quad \text{and}$$

$$U_q = U_o \Big/ \left(1 + C_f \frac{\rho S U_o}{2M}(\tau + T)\right) \tag{17}$$

and

$$X_q = (2M/\rho S)\ln\left(1 + C_f \frac{\rho S U_o}{2M} t_q\right). \tag{18}$$

Equations (12) and (13) can be calculated precisely:

$$t = (2M/\rho S)\sqrt{\rho/2p}\ln\left|\tan\left(\pi/4 + 0.5ar\cos 1\Big/\sqrt{1+y_q^2}\right)\Big/\right.$$
$$\tan(\pi/4 + 0.5ar\cos 1/\sqrt{1+y^2})$$
$$\left.\cdot\left[\frac{(\sqrt{1+y_q^2}-3)(3\sqrt{1+y^2}+\sqrt{8}y-1)}{(\sqrt{1+y^2}-3)(3\sqrt{1+y_q^2}+\sqrt{8}y_q-1)}\right]^{3/\sqrt{8}}\right| + t_q \tag{19}$$

and

$$x = x_q + (M/8\rho S)\ln\left|\left[\frac{y^6(\sqrt{1+y^2}-1)(\sqrt{1+y_q^2}+1)}{y_q^6(\sqrt{1+y_q^2}-1)(\sqrt{1+y^2}+1)}\right]\right.$$
$$\left.\times\left(\frac{\sqrt{1+y_q^2}-3}{\sqrt{1+y^2}-3}\right)^6\right|. \tag{20}$$

Together with eqns (2) and (3) when $t < t_q$, eqn (19), which establishes connection between the time and velocity of the particle motion, and eqn (20), which in turn establishes connection between the depth of penetration and the particle velocity when $t \geqslant t_q$, to give a full description of the process of solid particle penetration into the metallic target under the conditions of its loading by a dense high-speed flux of particles.

SECOND STAGE OF PENETRATION UNDER THE PRESSURE WHICH DEPENDS ON TIME

As can be easily seen from eqns (10)–(13), the suggestion that the pressure p generated in the target by a flux of particles must be constant is not a prerequisite one. If it is assumed that $p = p(t)$, the general form of eqns (12) and (13) will remain the same and these equations can also be calculated precisely. Moreover, under the condition that, when $t < t_q$, the change in pressure is rather small and can be considered constant (t_q is approximately equal to 10^{-7} s and the total time of the existence of pressure in a real experiment is usually equal to about 10^{-4} s, so that such an assumption is quite applicable for the case), the process will be totally described by eqns (15–20). Only the quantity p in eqns (10) and (20) will be replaced by $p(t)$ and eqn (19) will have the form:

$$\int_{t_q}^{t}\sqrt{2p(t)/\rho}\,dt = (2M/\rho S)\ln\left|\tan\left(\pi/4 + 0.5ar\cos 1\Big/\right.\right.$$
$$\left.\sqrt{1+y_q^y}\right)\Big/\tan(\pi/4 + 0.5ar\cos 1/\sqrt{1+y^2})$$
$$\left.\cdot\left[\frac{(\sqrt{1+y_q^2}-3)(3\sqrt{1+y^2}+\sqrt{8}y-1)}{(\sqrt{1+y^2}-3)(3\sqrt{1+y_q^2}+\sqrt{8}y_q-1)}\right]^{3/\sqrt{8}}\right|. \tag{21}$$

If the precise dependence $p = p(t)$ has been previously calculated or established experimentally, eqn (21) allows one to establish the dependence $U = U(t)$, eqn (20) describes the dependence $x = x(t)$ and together with eqns (2) and (3), when $t < t_q$, will form a complete solution of the problem for the case.

DISCUSSION

Figures 3 and 4 graphically demonstrate the change in time of the particle velocity and depth of penetration changing for two possible cases:

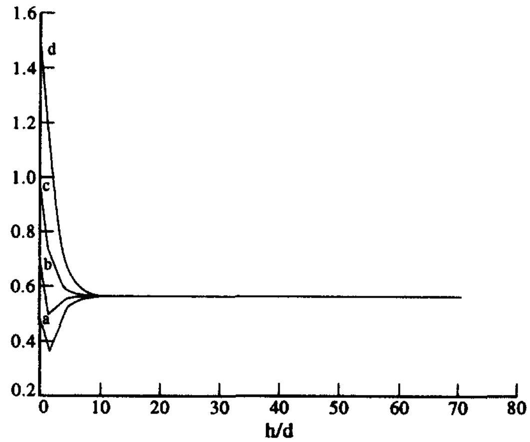

Fig. 3. The function $U(t)$ for the case of the penetration of W particles ($d = 10$ μm) into an iron target for different values of the initial velocity U_o under a constant pressure of about 10 GPa; (a) $U_o = 0.5$ km s^{-1}, (b) $U_o = 0.7$ km s^{-1}, (c) $U_o = 1.0$ km s^{-1}, (d) $U_o = 1.5$ km s^{-1}.

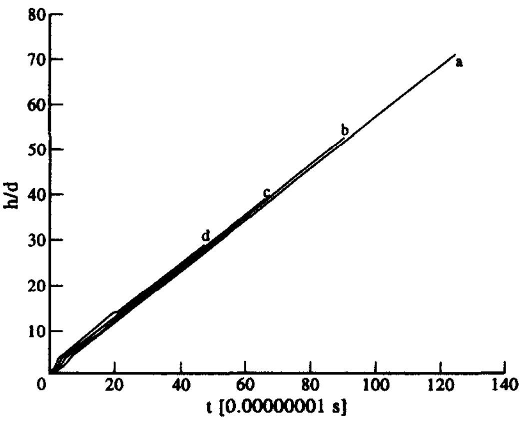

Fig. 4. The function of $x(t)$ for the case of the penetration of W particles ($d = 10$ μm) into an iron target for different values of the initial velocity under a constant pressure of about 10 GPa: (a) $U_o = 0.5$ km s^{-1}, (b) $U_o = 0.7$ km s^{-1}, (c) $U_o = 1.0$ km s^{-1}, (d) $U_o = 1.5$ km s^{-1}.

$$1.\ \sqrt{1+y_q^2}-3 \leqslant 0$$

$$2.\ \sqrt{1+y_q^2}-3 > 0. \qquad (22)$$

It can be easily seen that in both cases the particle velocity tends asymptotically to the value

$$U_{St} = 0.5\sqrt{p/\rho} \qquad (23)$$

and for $t \gg t_q$ the depth of the particle penetration is increased almost in complete correspondence with the law $x = U_{st}t$. For example, when $p = 10$ GPa for steel ($\rho = 7830$ kg m^{-3}), $U_{st} = 565.053$ m s^{-1}. Practically, the loading time is $t_f = 10^{-4}$ s, therefore, the ultimate depth of penetration that can be achieved by a particle in this case for steel is $H = 0.0561$ m, or, when $d = 10$ μm, $H/d = 5610$. Figures 5 and 6 show the results of calculations for the velocity of motion and depth of penetration of a tungsten particle in an iron target under SDP, when the pressure, generated in the target by a flux of particles, changes with time (the pressure curve is also depicted in both figures).

The SDP model presented has some limitations imposed on the parameters of the flow and particles. It is obvious that the model can be valid only if $t_c \ll t_f$, that is

$$d \ll 2t_f\sqrt{p/\rho} \qquad (24)$$

for a steel target this condition means that $d \ll 0.1$ m, which is quite satisfied when $d < 10^{-4}$ m. There is also a restriction on the particle velocity, since the model uses the concept of an inviscid fluid in relation to the target material flowing around the particle. Otherwise, it would be necessary to take into account a viscous share of the hydrodynamic resistance force when

$$Re = Ud/\mu \leqslant 10 \qquad (25)$$

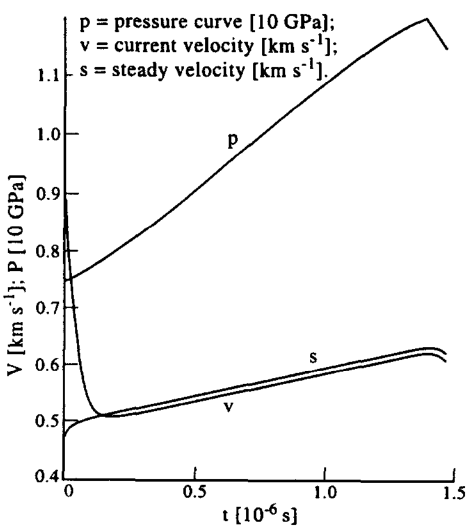

Fig. 5. The velocity–time curve for a varying pressure. p, the pressure time curve, 10 GPa; v, the current velocity [eqn (21)] vs time, km s^{-1}; s, the stable velocity [eqn (23)] vs time, km s^{-1}.

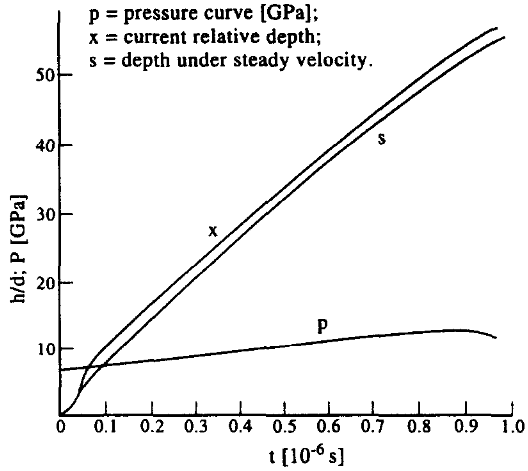

Fig. 6. The depth of penetration vs time curve for varying pressure. p, the pressure time curve, 1 GPa; x, the current depth of penetration [eqn (20)] vs time; s, the depth of penetration under a stable velocity, defined by eqn (23) vs time.

which means that under SDP in accordance with the model presented

$$U_q \leqslant 10\ \mu/d \quad \text{and} \quad U_{St} \leqslant 10\ \mu/d \qquad (26)$$

where μ is the target viscosity and U_q was defined by eqn (17). For steel this means that U_q and U_{st} should exceed 25 m s^{-1} ($d = 10$ μm). For steel this is equivalent to the requirement that the pressure p generated in the target by powder flux should exceed 0.07 GPa.

CONCLUSION

The hydrodynamic model of SDP presented allows one to obtain a precise solution of the problem of penetration of solid axisymmetric particles into a metallic target under the conditions corresponding to the requirements of SDP experiment. It was established that for the realisation of SDP the following special conditions are required:

(1) The target material in the region of SDP must be unhardened by impulse treatment of the target by a dense high-speed flux of particles [1–3];
(2) A flux of particles has to be dense enough and have a high speed in order to create in the target a pressure field substantially greater than 0.1 GPa. Via the mechanism of the formation of a high-velocity jet on closure of the channels formed in the target by penetrating particles, the potential energy of this pressure field is transmitted to the particles and compensates their energy losses in penetration.

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